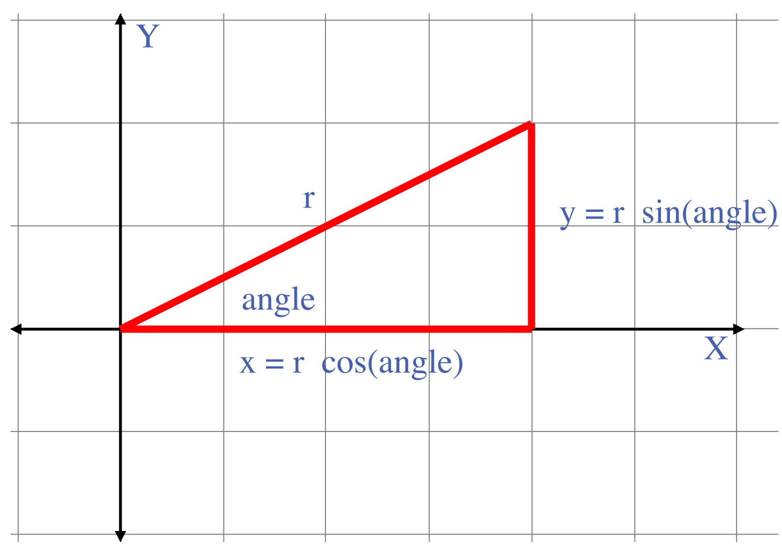
Polar and Rectangular (Cartesian) Coordinate Conversion

Robert B. Heckendorn
University of Idaho

April 29, 2015

1 Angle and Distance to X and Y

Conversion from an angle and a distance to the X and Y position at that distance and angle is very easy thanks to the sine and cosine functions from trigonometry. All right triangles with a given angle are **similar**. That is, the ratios of their sides are the same from one triangle to the next. This is because if the angles are fixed, increasing the length of one side makes the other sides grow proportionally. If you give me an angle θ I can tell you that the ratio of the "side opposite the angle" to the "long side" of the triangle, called the hypotenuse without having to draw the triangle. That ratio is the sine of the angle denoted: $\sin(\theta)$. Similarly for the cosine, denoted $\cos(\theta)$. The rules for the conversion from angle and distance, r, can be found in this diagram:



The rule is $x = r\cos(angle)$ and $y = r\sin(angle)$.

Simple.

2 X and Y to Angle and Distance

The same kind of argument holds for converting back to the length that was r in the first diagram. This is the length of the hypotenuse and so $r = \sqrt{x^2 + y^2}$. From trigonometry we know the tangent of the angle is the length of the "side opposite of the angle" divided by the length of the "side adjacent to the angle". This is written $\tan(angle) = y/x$. arctan, also known as atan, is the inverse of the tan function. So we have $angle = \arctan(\tan(angle)) = \arctan(y/x)$ as in the next diagram.

Further reading: https://www.processing.org/tutorials/trig/